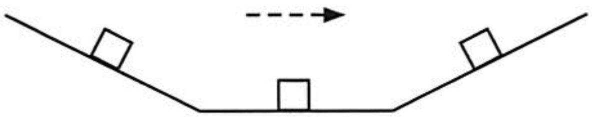
ABRHS PHYSICS (H) NAME: KEY

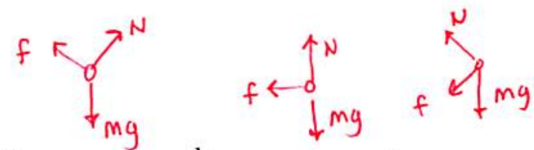
Friction & Rolling Without Slipping

Rolling without slipping is an extremely useful special condition for objects that that move and rotate simultaneous. That is because it connects the velocity of the center of mass with the rotational velocity of the object via $v = r\omega$. However, understanding frictional forces during rolling without slipping can be very tricky.

Before we do rolling, first imaging a box sliding down an incline or on level ground. In the diagram below, the three situation are shown. In all three cases the object moves to the right, so in (a) the object is moving down the incline, in (b) the object slides to the right and in (c) the object goes up the incline. For each case draw the appropriate free body diagram.



a b c



a. b. c.

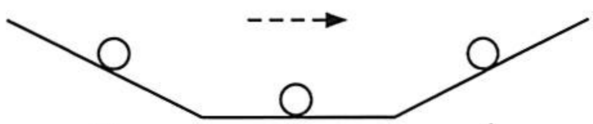
1. Those were hopefully pretty easy as they all involved *kinetic* friction. How do you tell the direction of kinetic friction?

Always opposes the motion & always tries to stop the relative motion

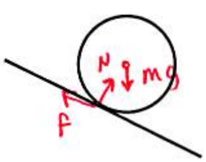
2. Now for the rolling situation. When an object is undergoing "rolling without slipping" what type of friction is present - static or kinetic? How do you know?

static! Because the contact point with the ground is not moving.

3. The diagram below shows an object rolling without slipping. In (d) the object is rolling down the incline, in (e) it is rolling to the right and in (f) it is rolling up the incline. For each of the situations, draw the appropriate force diagram in the space below

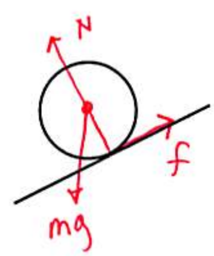


d e f

d) 

e) 

f=0!

f) 

4. In general, how do you tell the direction of *static* friction?

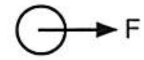
static only does what it has to (to a certain maximum possible value) to keep the 2 surfaces from sliding across each other.

Friction & Rolling Without Slipping

Before we go to one last rolling without slipping problem, let's do a quick review of Newton's Laws. The next few problems all involve a round object of mass m, radius r, and moment of inertia I.

always: $\Sigma F = ma$ & $\Sigma \tau = I\alpha$

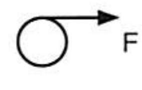
5. Now imagine there is a constant force F acting through the center of mass of the object. There are no other forces on the object. Write out Newton's Second Law for both translation and rotation and explain what will happen to the object.



$F = ma$ & $\tau = 0$

~~obj~~ object just accelerates to right. No rotation.

6. Now imagine there is a constant force F acting from the top edge of the object (as if it was from a string wrapped wound the object.) There are no other forces on the object. Write out Newton's Second Law for both translation and rotation and explain what will happen to the object.



$F = ma$

and $rF = I\alpha$

But "a" and "α" have no set relationship and are independent of each other.

Object accelerates to right and starts rotating ↻

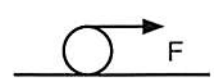
7. Now imagine there is a constant force F acting from the bottom edge of the object (as if it was from a string wrapped wound the object.) There are no other forces on the object. Write out Newton's Second Law for both translation and rotation and explain what will happen to the object.

$F = ma$ & $rF = I\alpha$

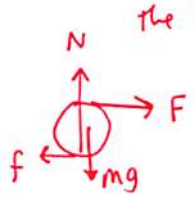
same as #6, but rotates opposite way.

This one is actually pretty tricky!

8. Now imagine that object is on a level surface and rolls without slipping while being pulled with a constant force acting through the top edge as shown. ~~Write out Newton's Second Law for both translation and rotation. What other relationship do you know in this situation?~~



→ It turns out, friction will be to the right (probably) so let's assume its to the left and see what happens.



only ⟷ matters, so N2L:

$\Sigma F = ma$ $\Sigma \tau = I\alpha$ & roll without slipping

① $F - f = ma$ ② $rF + rf = I\alpha$ & ③ $a = r\alpha$

↳ $F = f + ma$ ↳ $F + f = \frac{I}{r}\alpha$ ↳ $\alpha = \frac{a}{r}$

substitute ① into ②

$$(f + ma) + f = \frac{I}{r}\alpha$$

$$2f = \frac{I}{r}\alpha - ma = \frac{Ia}{r^2} - ma$$

Now, let's say $I = kmr^2$ where "k" is some constant (a fraction)

So $2f = \frac{kmr^2 a}{r^2} - ma = kma - ma$

So $f = \frac{1}{2}(k-1)ma$

Notice k is a fraction so that is negative! ∴ f is to the right! *

* Notice a hoop has k=1, if a hoop, f=0!